

From Student Invented Strategies to Standard Algorithms: What's the Rush?

VCTM

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Fran Huntoon

franhuntoonvt@gmail.com

Algorithms

What is an algorithm?

Simply put, an algorithm is a set of rules for solving a particular kind of problem.

"Building Support for School Mathematics: Handbook for Working with Parents and the Public by R. Parker and J. Akers-Mitchell, Heinemann, 2006

An algorithm consists of a precisely specified sequence of steps that will lead to a complete solution for a certain class of computational problems. Computational Fluency, Algorithms, and Mathematical Proficiency: One Mathematician's Perspective" by Hyman Bass in *Teaching Children Mathematics*, February, 2003, Volume 9, Issue 6

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. CCSSM Glossary

History of algorithms

Muhammad ibn Mūsā al'Khwārizmī, a ninth-century Persian mathematician. His latinized name, Algoritmi, meant “the decimal number system” and was used in this meaning for centuries. Kowalkiewicz, M. (2019, Oct. 10)*How did we get here? The story of algorithms.*

Although there is some evidence of early multiplication algorithms in Egypt (**around 2000-1700 BC**), the oldest written algorithm is widely accepted to have been found on a set of Babylonian clay tablets that date to around **1800-1600 BC**.

What Are Student Invented Strategies?

Carpenter and colleagues (1998) refer to any strategy other than the standard algorithm or that does not involve the use of physical materials or counting by ones as *invented strategy*.

In the Common Core State Standards (CCSSO, 2010) they describe these in their expectations for first and second graders as “strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.”

Van de Walle, John A. (2013). Elementary and middle school mathematics : teaching developmentally. Boston :Pearson,

$$\begin{array}{r} \\ \\ + \\ \hline 7 9 7 \end{array}$$

Student Invented Algorithms

Teachers had to consider whether the procedure was (1) *efficient* enough to be used regularly without considerable loss of time and frustration due to the number of recorded steps required. Student invented algorithms also had to be (2) *mathematically valid* and (3) *generalizable*. These three criteria became the “standards” for algorithms. Campbell, P.F., Rowan, T.E., Suarez, A.R. (1998) What criteria for student invented algorithms? The teaching and learning of algorithms in school mathematics

Student Invented Strategies vs. Algorithms

- Invented strategies are number oriented rather than digit oriented.
- Invented strategies are left-handed rather than right-handed.
- Invented strategies have a range of flexible options rather than “one right way.”
- student-centered rather than teacher-directed and,
- focused on number sense and mental math

Van de Walle, John A. (2013). Elementary and middle school mathematics : teaching developmentally. Boston :Pearson,

$$200 + 35 + 45 = 280$$

A handwritten diagram illustrating the addition of 200, 35, and 45 to get 280. The numbers are arranged in a triangular shape. Lines connect the digits: a line from the '2' of 200 to the '2' of 280; a line from the '0' of 200 to the '0' of 280; a line from the '3' of 35 to the '8' of 280; a line from the '5' of 35 to the '0' of 280; a line from the '4' of 45 to the '8' of 280; and a line from the '5' of 45 to the '0' of 280. There are also two small circles drawn in the middle of the diagram, one between the '3' and '4' and another between the '5' and '5'.

There are 16 players on each soccer team in the Smithville Soccer League.

A. How many players are in the league if there are 12 teams?

Show your work

$$10 \times 12 = 120$$

$$6 \times 12 = 72$$

192 on 12 teams

B. How many players are in the league if there are 24 teams?

Show your work.

$$12 + 12 = 24$$

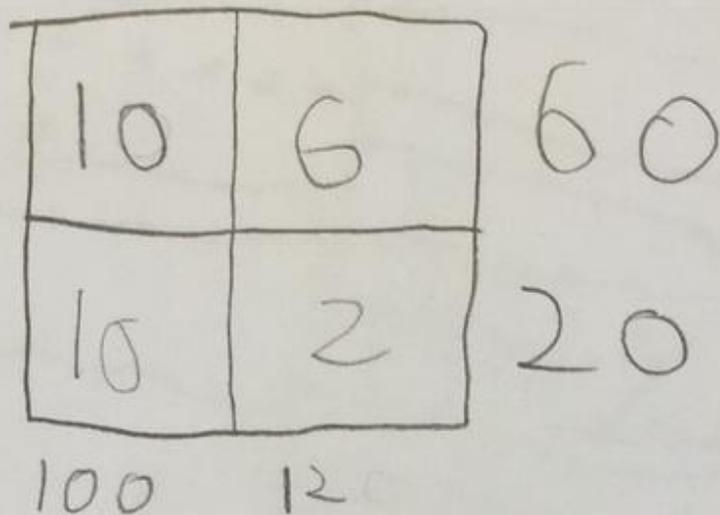
$$192 + 192 = 384 \text{ on } 24 \text{ teams}$$

There are 16 players on each soccer team in the Smithville Soccer League.

A. How many players are in the league if there are 12 teams?

Show your work

$$\begin{array}{r} 112 \\ + 80 \\ \hline 192 \end{array}$$



There are 192 players in all.

Algorithms in the CCSSM

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Algorithms in the CCSSM

Number and Operations in Base Ten

5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.

The Number System

6.NS

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Qualities of Algorithms

- Accuracy (or reliability)
- Generality
- Efficiency
- Ease of accurate use
- Transparency

Computational Fluency, Algorithms, and Mathematical Proficiency: One Mathematician's Perspective" by Hyman Bass in *Teaching Children Mathematics*, February, 2003, Volume 9, Issue 6

Ben's Method

$$\begin{array}{r} 358 \\ + \underline{163} \end{array}$$

Benefits of Student Invented Strategies

- Students make fewer errors.
- Less reteaching is required.
- Students develop number sense.
- Invented strategies are the basis for mental computation and estimation.
- Flexible methods are often faster than standard algorithms.
- Algorithm invention is itself a significantly important part of doing mathematics.
- Invented strategies serve students well on standardized tests.

Van de Walle, John A. (2013). Elementary and middle school mathematics : teaching developmentally. Boston :Pearson,

Alice ran a race in 50 minutes.

It took Dan $1\frac{1}{2}$ times longer to run the same race.

How long did it take Dan to run the race?

Show your work.

$$\begin{array}{r} 50 \\ \times 1 \\ \hline 50 \end{array} + \begin{array}{r} 50 \\ \times \frac{1}{2} \\ \hline 25 \end{array} = 75 \text{ mins}$$

It took
Dan 75 minutes
to complete the
race.
(1 hour and 15 minutes)

A string was 56 inches long. Dylan cut some off. Now the string is 27 inches long. How much of the string did Dylan cut off?
Show how you know.

$$56 - 27 = \square$$

$$50 - 20 = 30$$

$$6 - 7 = -1$$

$$30 - 1 = 29$$

29 inches of
string was cut
off.

$$\begin{array}{r} 3) 6,418 \\ -1,305 \\ \hline \end{array}$$

$$5,113$$

$$6,000 - 1,000 = 5,000$$

$$400 - 300 = 100$$

$$18 - 5 = 13$$

$$5,000 + 100 + 13 = 5,113$$

The Champlain Music Festival will be held in an area where there is no parking. This means that all 2,881 participants will have to take a shuttle bus from the parking lot to the festival site.

If a shuttle bus can transport 67 riders each trip, how many trips will it take to get all the participants to the festival site?

Show your work.

$$\begin{array}{r} 67 \\ \times 2 \\ \hline 134 \end{array}$$

$$\begin{array}{r} 67 \\ \times 20 \\ \hline 1340 \end{array}$$

$$\begin{array}{r} \overline{202021} \\ 67 \overline{) 2,881} \\ \underline{1,340} \\ 1,541 \\ \underline{1,340} \\ 201 \\ \underline{134} \\ 67 \\ \underline{-67} \\ 0 \end{array}$$

Answer
202021

~~$$\begin{array}{r} 67 \\ \times 10 \\ \hline 670 \end{array}$$~~

$$\begin{array}{r} 1 \\ + 67 \\ + 67 \\ \hline 134 \end{array}$$

Farmer Brown donated 7 dozen eggs to the senior center.

How many eggs did she donate?

Show your work.

$$7 \times 12 = ? = 12 \text{ eggs}$$

$$11 \times 7 = 77 + 7 = 84$$

$$7 \times 10 = 70 + 14 = 84$$

$$7 \times 12 = 84$$

warning

Ms. Emerson needed 82 juice boxes for a class picnic. The juice boxes come in packages of 6.

How many packages should Ms. Emerson buy to have enough for the class picnic?

$$6 \times 14^{\text{R2}} = 82$$

Student Invented Strategies: In the Classroom

Research indicates that students can succeed in inventing their own methods for solving basic computational problems (Madell, 1985; Kamii & Joseph, 1988; Cobb & Merkel, 1989; Resnick, Lesgold, & Bill, 1990; Carpenter, Fennema, & Franke, 1992). Inventing procedures flourishes when:

- the classroom environment is accepting and supportive;
- adequate time for experimentation is allotted;
- computational tasks are embedded in real-life contexts; and
- students discuss their solution strategies with the teacher and with one another.

Resources

Van de Walle, John A. (2013). Elementary and middle school mathematics : teaching developmentally. Boston:Pearson

Bass, H. (2003) Computational fluency, algorithms, and mathematical proficiency: one mathematician's perspective. *Teaching Children Mathematics*, 9, (6), 322 – 327.

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Number and Operations in Base Ten*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Fuson, K.C., & Beckmann, S. (2012 - 2013). Standard algorithms in the common core state standards. *NCSM Journal*, 14 (2), 14 – 30.